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The critical state in a random 3D Josephson net created by transport current

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Abstract
The set of weak links connecting superconducting grains of ceramics forms a random three-dimensional Josephson net. The temperature and size dependences of the critical currents in bismuth-based and yttrium-based high-temperature superconducting ceramic samples consisting of randomly oriented grains have been studied in zero magnetic field. It is shown that the critical current in samples having various cross-sectional shapes can be presented as the product of temperature and size dependent factors. The transport critical current is a homogeneous function of the cross-sectional shape dimensions. The size dependent factor is a universal function describing properties of the weak link Josephson net. The exponent of this function is independent of the temperature, the type of superconducting material, the existence (or lack) of a Josephson net in the sample, and the method used for varying the sample cross-section. The transport critical current density and induced magnetic field are homogeneous functions of coordinates. It is shown that the critical current density has a power law dependence on the magnetic field.

1. Introduction

High-temperature superconducting (HTSC) ceramics are polycrystalline materials consisting of superconducting grains with all possible orientations. The passage of a transport current through this material is possible due to the weak links. The intra-grain critical current in ceramics is much greater than the inter-grain critical current [1]. Link properties, impurity segregation, shape, and mutual orientation of grains are the causes of a sharp decrease of transport critical current. Several models have bean proposed for explaining the critical current flow in ceramics. If no texturing or special aligning processes are used, then ceramics consisting of superconducting grains with all possible orientations represent the three-dimensional random Josephson network of weak links [2–4].

In the absence of an external magnetic field, the medium surrounding the sample is isotropic. This considerably simplifies the system under study and makes it possible to elucidate a number of its important properties. Nevertheless, magnetic field induced by the transport current is present in the system. The distribution and value of the transport critical current density in the system are determined by the magnetic field. The distribution of this field in the sample depends on the critical current value and cross-sectional shape of the sample. As a result, a size effect is observed: the dependence of the critical current I_c on the size of the sample cross-section. Many authors have hence investigated a size effect [4–11].

The purpose of the present work is to investigate various 3D Josephson nets composed of randomly oriented superconducting grains. To accomplish this goal, we investigated the critical current in Y- and Bi-based ceramics. The induced field can be controlled by varying the transverse sizes of the sample. We have used this variation as a tool for studying the critical state in Josephson networks. Our results indicate that the critical current, its density, and its induced magnetic field in the network obey the scaling law.

2. Experimental details

The samples used were prepared from high-purity oxides and carbonates. Mixed components were powdered, pressed and calcined in air. Then the tablets were ground up, pressed and

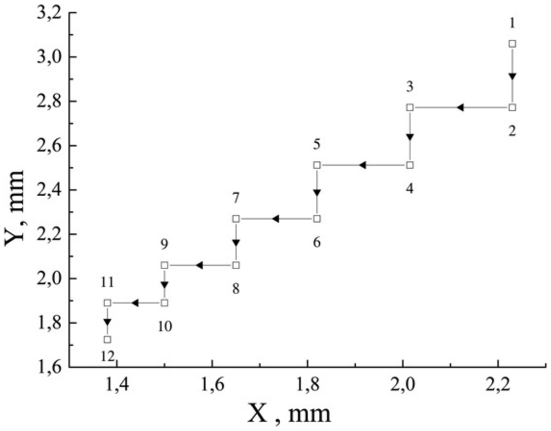

Figure 1. Diagram illustrating the variation of the cross-section of sample 1.

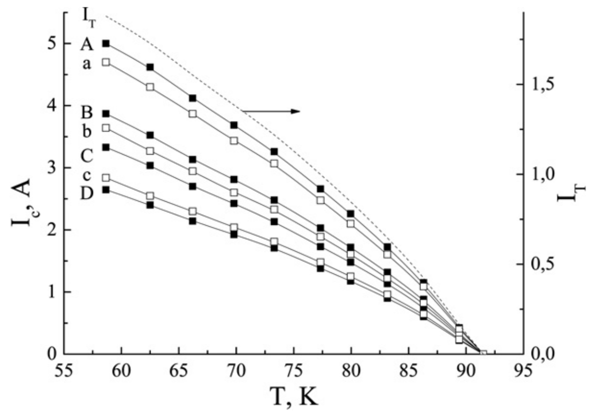

Figure 2. Temperature dependence of the critical current in sample 1 for two families of cross-sections. Lines A, B, C, D correspond to sections (in mm^2) 2 (2.23 × 2.75), 6 (1.8 × 2.29), 8 (1.63 × 2.04), 12 (1.38 × 1.72), and lines a, b, c correspond to sections 3 (2.02 × 2.75), 7 (1.63 × 2.29), 11 (1.38 × 1.89), respectively. $T_0 = 77.33$ K.

calcined in air a few times. No texturing or special oxidation processes were used. Some details are given in [12].

The samples under study were in the form of toroidal rings. The rings were machined with a lathe using special lathe tools. Sample 1 (Y-123) and sample 2 (Bi-2223) had rectangular cross-sectional shapes. Sample 3 had a trapeziform cross-sectional shape and contained two superconducting phases (Bi-2212 and Bi 2223). Using a two-phase sample permitted the study of critical currents in two different Josephson nets in the same sample to be carried out. The superconducting transition temperatures (T_c) of samples 1, 2, 3 were 91 K, 106 K, and 99 K respectively.

The critical current was measured using the contactless transformer technique applied by many researchers to study high-temperature superconductors [7, 8, 13–15]. It must be noted that I_c was measured in the absence of an external field. In our experiments, a ring-shaped sample, along with the primary winding (with n_1 turns) and measuring winding, was placed in a pot ferrite core. As the value of the primary alternating current I_1 (14 Hz) grew, the current in the ring varied so as to keep the magnetic flux accumulated by the central limb of the core and closing through the lateral and end walls constant and equal to zero. Consequently, the magnetic induction and magnetic field within the pot core remained zero. Therefore, the sample was subjected only to the magnetic field of the self-current. Individual parts of the ring were each screened by the central limb of the core against the fields produced by the current in other sections of the ring. At the time when the current in the ring reached a critical value, a signal in the form of a sharp peak was induced in the measuring winding. During the measurements, this peak was kept as low as possible. At the instant when the peak was reduced to zero on varying the primary current, the value of I_1 was recorded and I_c was found, $I_c = n_1 I_1$. More details are given in [7, 13–15]. The critical current in the sample having the initial size was measured at a number of fixed temperatures. Then the sample was lathed and the critical current was measured at the same temperatures.

The same technique was used for the critical temperature determination. Only a sinusoidal signal was induced in the secondary coil when the temperature was higher than the critical temperature. A distorted waveform showed up, in the form of a coat-hanger, when the decreasing temperature became slightly less than T_c (at constant amplitude of I_1). The appearance (or disappearance with increasing temperature) of this waveform indicated that the critical temperature was reached. The measuring current density in the sample was always less than 10 mA cm^{-2}.

The height (Y) and width (X) of sample 1 were varied one after another (see figure 1). As a result two similar cross-section families for the sample were obtained. For the first family (odd-numbered sections, from 2.23 × 3.06 to 1.38 × 1.89 mm^2) the average value of the ratio X/Y was equal to 0.728 and for the second family (even-numbered sections, from 2.23 × 2.772 to 1.38 × 1.725 mm^2), $X/Y = 0.8$. The critical current in this sample was measured from 58.6 K to T_c (11 isotherms). The width of sample 2 was changed from 2.3 to 0.25 mm (13 cross-sections) at constant height ($Y = 1.5$ mm), and the critical current in the sample was measured from 66.25 K to T_c (10 isotherms). Sample 3 was studied in the temperature range from 4.2 K to T_c for ten cross-sections. Initially the median (X) of the trapezoid was varied from 2.56 to 1.02 mm at constant height Y (2.5 mm). Then the height was varied up to 0.55 mm at constant median.

3. Results and discussion

Temperature dependences of the critical currents in samples 1–3 are shown in figures 2–4. Samples 1 and 2 consist of single-phase material. The critical currents in the samples increase smoothly as the temperature decreases. In sample 3 a superconducting transition in grains of Bi-2212 phase takes place at temperatures below 80 K. A new Josephson net formed by two phases, Bi-2223 and Bi-2212 advents. As a result, an increase in curvature of $I_c(T)$ is noticed with decreasing temperature. The cross-sections of this sample are not similar trapezia, but the $I_c(T)$ curves are similar to each other.

It will now be seen that the expression for the critical current can be written in the form $f(T)G(X, Y)$ where

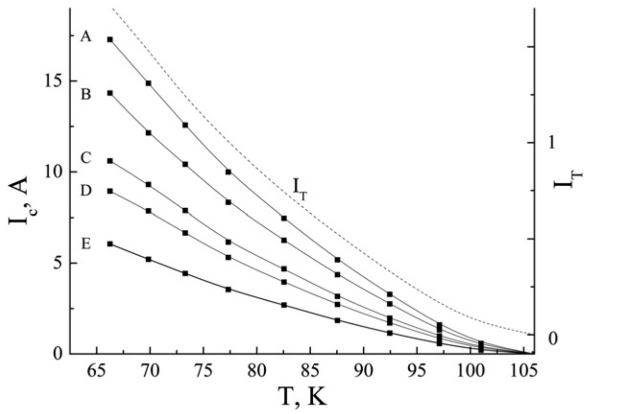

Figure 3. Temperature dependence of the critical current in sample 2 for cross-sections having various base widths (mm): 2.3 (A); 1.74 (B); 1.1 (C); 0.86 (D); 0.49 (E). $T_0 = 77.33$ K.

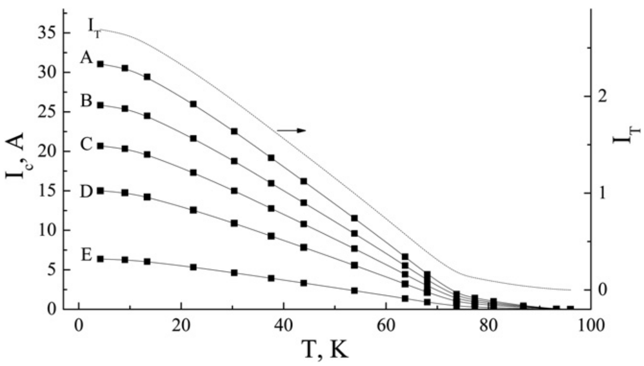

Figure 4. Critical current in sample 3 as a function of the temperature at constant height (2.5 mm) and various medians (in mm): 2.3 (A), 1.78 (B), 1.28 (C); and at constant median (1.02 mm) and various heights (in mm): 1.2 (D), 1.78 (E). $T_0 = 53.8$ K.

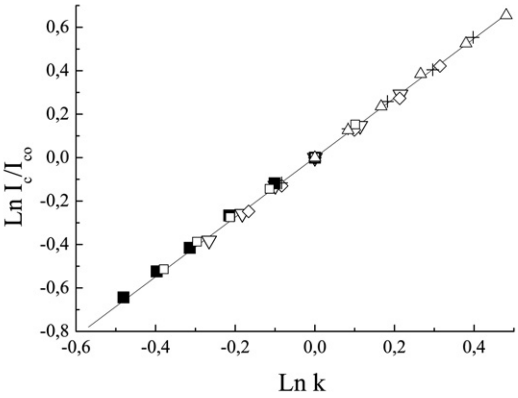

Figure 5. Dependence of the relative critical current in sample 1 on the relative size of the sample cross-section for different temperatures. Odd sections: 83.15 K (□), 73.3 K (▽), 66.2 K (◇), 58.65 K (+). Even sections: 86.3 K (■), 62.5 K (△). The quantities corresponding to odd (3, 5, 7, 9) and even (2, 12) cross-sections (see figure 1) were used as normalization values (i.e. X_0, Y_0, I_0) for data at the indicated temperatures.

$G(X, Y)$ is a homogeneous function. Let us introduce the relative current I_T by dividing the critical current experimental values of one of the $I_c(T)$ curves by the value of the critical current determined for the same section at temperature T_0 (T_0 is one of the experimental study values of T):

$$I_T = I_c(X, Y, T)/I_c(X, Y, T_0). \qquad (1)$$

The values of I_T obtained for different cross-sections of each sample are so close that the corresponding points in figures 2–4 cannot be distinguished on the scale adopted. For this reason, the dashed curves in the figures are the smoothed curves drawn through the mean values. The function $I_T(T)$ goes to zero as the increasing temperature tends to T_c and its curve is similar to the $I_c(T)$ curve of each sample. Like the $I_c(T)$ curves, the function $I_T(T)$ for sample 3 has a bend associated with the superconducting transition in the Bi-2212 phase. Therefore, the function $I_T(T)$ depends on specific properties of the sample only and is independent of the cross-sectional shape and dimensions.

Similar cross-sections of sample 1 yield two families of sections. It would be interesting to find variations of the critical current corresponding to these cross-sections. In other words, we are interested in the dependence of the ratio $I_G = I_c(X_i, Y_i, T)/I_c(X_0, Y_0, T)$ on k_i, $k_i = X_i/X_0 = Y_i/Y_0$. Index i is a section number (see figure 1). Experimental results corresponding to the cross-sections marked 0 were used as normalization values. The dependence of I_G on k is shown on figure 5 on a logarithmic scale for six isotherms of two section families. The points may be all described using the linear dependence

$$\ln I_G = p \ln k.$$

Function $I_G(k)$ does not depend on the choice of normalization values. Another choice of normalization values results in the same curve. So we have

$$I_c(kX, kY, T) = k^p I_c(X, Y, T). \qquad (2)$$

In the absence of an external magnetic field the critical current in the random three-dimensional Josephson net of weak links existing in HTSC ceramics follows a homogeneous Euler function [16] with exponent p. Evaluating exponent p for each isotherm of sample 1 we obtain a set of values coinciding within confidence intervals. Hence these values can be averaged. As a result we obtain $p = 1.37 \pm 0.01$. Here and below, confidence intervals are determined by using quantiles of the Student distribution with a confidence probability of 0.95. Thus, the exponent of the Euler function does not depend on the temperature, and hence the relative current I_G does not depend on temperature either and is determined only by the geometrical arguments:

$$I_G(X, Y) = I_c(X, Y, T)/I_c(X_0, Y_0, T). \qquad (3)$$

Since the critical current in our sample can be presented as (1) and (3), it has the form

$$I_c(X, Y, T) = f(T)G(X, Y). \qquad (4)$$

Function $f(T)$ depends on the temperature and specific properties of each sample only. $G(X, Y)$ depends on the

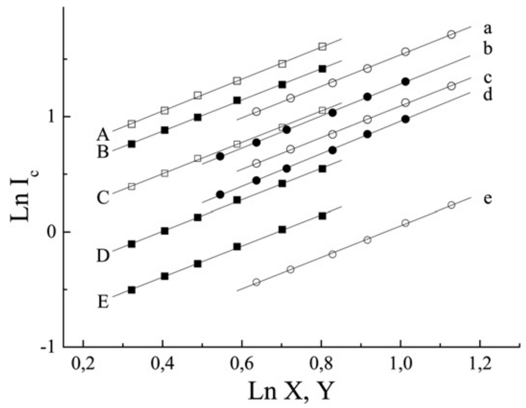

Figure 6. Dependence of the critical current in sample 1 on the sample width (X) and height (Y) for different temperatures. Open and solid symbols correspond to odd and even sections in figure 1. Lines denoted by A–E correspond to the dependence on X at temperatures (K): 62, 66, 77, 83, 86. Lines denoted by a–e correspond to the dependence on Y at temperatures (K): 58, 69, 73, 77, 86.

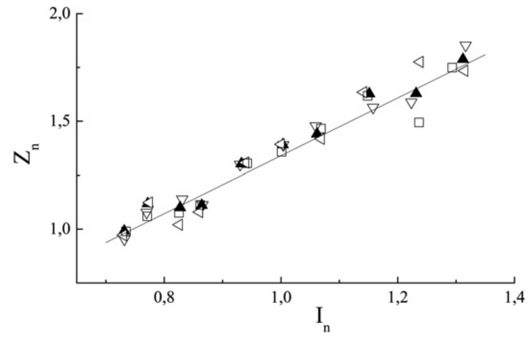

Figure 7. Correspondence of experimental data obtained for sample 1 to the Euler equation (equation (6)). The standardized values Z_n and I_n are used (see the text). □, ◁, ▽, ▲ correspond to isotherms 58.65, 73.3, 83.15, 89.4 K.

sample cross-sectional sizes. Equations (2) and (4) show that $G(X, Y)$ is a homogeneous Euler function. $G(X, Y)$ is the symmetrical function of X and Y. This fact follows from the absence of the external field. It makes no difference which side of the cross-section is taken as the width (X) and which the height (Y). Now we shall see that the critical current in a random three-dimensional Josephson net not only corresponds to the definition of a homogeneous function, but also is in accord with all properties of such a function. The critical current $I_c(X, Y, T)$ must satisfy the equations [16]

$$I_c(X, Y, T) = X^p F_1(Y/X), \quad I_c(X, Y, T) = Y^p F_2(X/Y). \tag{5}$$

In the case of two variables, only such functions are homogeneous. Since the ratio of the sides for each family of cross-sections of sample 1 is constant, functions $F_1(Y/X)$ and $F_2(X/Y)$ depend on temperature only. Figure 6 shows the dependence of the critical current in sample 1 on X and Y in double-logarithmic coordinates. It may be seen that our experimental results are in accord with equation (5). Curves I_c may be all approximated by a common power function, $I_c \sim X^p$ or Y^p. The evaluation of exponent p for each isotherm and each cross-section family gives a set of values coinciding within confidence intervals (confidence probability 0.95). The deviations of the exponent values from the average value are of random nature. So the exponent is independent of the temperature. Index p is equal to 1.37 ± 0.01.

Let us prove that the critical current in our sample satisfies the Euler differential equation [16]

$$X(\partial I_c/\partial X) + Y(\partial I_c/\partial Y) = pI_c. \tag{6}$$

That is, the function of the derivatives on the left-hand side must be proportional to the critical current and the factor p must coincide with the value obtained earlier. The experimental results obtained for sample 1 make it possible to determine the derivatives for sections from 2 to 11 by using the difference method (see figure 1). The results for four isotherms are shown on figure 7. The left-hand side of equation (6) is denoted by Z. In order to demonstrate the results obtained at various temperatures the standardized values Z_n and I_n are used for each isotherm. $Z_n = Z/Z_0$, $I_n = I_c/I_0$, Z_0 and I_0 are intermediate values of Z and I_c at a given value of the temperature. Figure 7 shows that our results satisfy the Euler differential equation. Z is a linear function of the critical current. The value of the factor p obtained for all isotherms is equal to 1.34 ± 0.04. This value coincides with the one obtained earlier. The larger confidence interval is associated with the errors arising from the evaluation of partial derivatives. Hence the critical current in a random three-dimensional Josephson net satisfies the definition of the homogeneous function and all its properties. Note that the same experiment was performed earlier [14]. However, in [14] the Bi2223 sample was studied at 77.33 K only, and the confidence intervals were considerably wider.

Thus function $G(X, Y)$ in equation (4) is the homogeneous function and independent of temperature. It is this function that contains information on the critical state in a Josephson net. Therefore, it is very important to determine the function $G(X, Y)$ in every way. To this end we performed a special comparative experiment. X (or Y) was varied individually. Figures 8 and 9 show the dependence of the critical current in samples 2 and 3. The width of the rectangular sample 2 was changed at constant height. The median of the trapezoid sample 3 was varied at constant height and then the height was varied at constant median. As before, relative currents I_G (dashed curves in figures 8 and 9) are independent of the temperature and choice of the normalization values. Another choice of normalization values only displaces the I_G curve. Thus, the critical current can be presented as equation (1) or (3). This means that the experimental values of the critical current satisfy equation (4). It is necessary to state that the cross-sections of these samples are not similar, but isotherms $I_c(X)$, $I_c(Y)$ and curves I_G for each sample look alike and smooth. No one cross-section is preferred. The curves may be approximated by a common power function X^r or Y^r:

$$I_G = G(X, Y)/G(X_0, Y) = (X/X_0)^r, \tag{7}$$

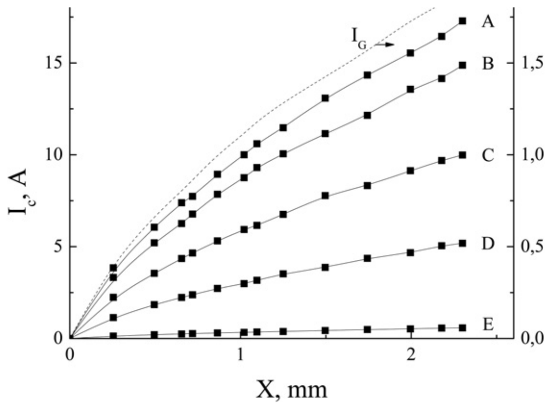

Figure 8. Temperature dependence of the critical current in sample 2 on the widths of rectangular cross-section at various temperatures. Lines A–E correspond to 66.25, 69.85, 77.33, 87.55, 101 K.

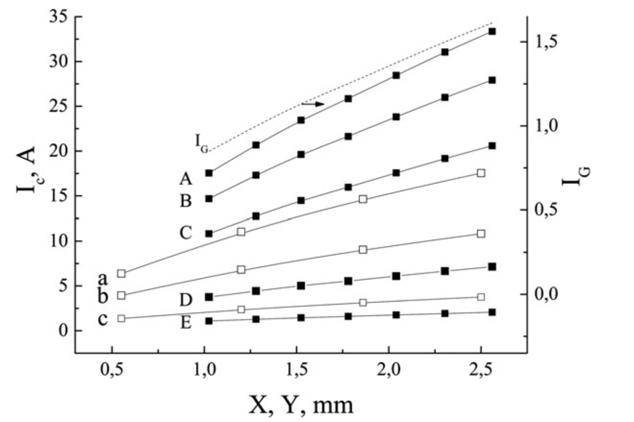

Figure 9. Temperature dependence of the critical current in sample 3 on the median (X, lines A–E) and the height (Y, lines a–c) of the trapezoid cross-section at various temperatures. Lines A–E correspond to 4.2, 22.4, 37.6, 63.7, 73.3 K. Lines a–c correspond to 4.2, 37.6, 63.7 K.

$$I_G = G(X, Y)/G(X, Y_0) = (Y/Y_0)^r. \quad (8)$$

Evaluating exponent r for samples 3 for each isotherm at each fixed X_0 and Y_0 we obtain a set of values coinciding within confidence intervals. The exponent value deviations from the average value are of random nature, i.e., the exponent is independent of temperature as well as the method of cross-section change. The average value of r is equal to 0.69 ± 0.02 and close to $p/2$. In the case of sample 2, using equation (7) we obtain $r = 0.68 \pm 0.01$. The external field was absent in our experiments, and so variables X and Y are absolutely equivalent. Therefore, if we had changed the height (Y) of sample 2 rather than X, we would have obtained the same result, i.e. equation (8). Using equations (7) and (8), we have $G(X, Y) = G(X_0, Y_0) \cdot (XY/X_0Y_0)^r$. X_0 and Y_0 are arbitrary and hence

$$G(X, Y) = AX^rY^r \qquad \text{or} \qquad G(X, Y) = AX^{2r}(Y/X)^r. \quad (9)$$

It is obvious that $G(X, Y)$ is a homogeneous function of X and Y, $r = p/2$, and we can write

$$I_c(X, Y, T) = Af(T)X^{p/2}Y^{p/2}. \quad (10)$$

Let us derive this equation from analytical consideration. Let us have a long sample having rectangular or trapezoid cross-section along the full length. The ratio Y/X of the left part of the sample may be arbitrary, but for right part $X = Y = X_0$. Let I_c and I_{c0} be the critical currents of the left and right parts of the sample. Varying the sides X, Y, X_0 we can reach the state when

$$I_c(X, Y, T) = I_{c0}(X_0, T). \quad (11)$$

Using equations (4), (5) we can rewrite equation (11):

$$X^pF_1(Y/X) = X_0^pF_1(1). \quad (12)$$

The properties and temperature of the sample are assumed to be identical along the full length and so functions $f(T)$ canceled. If we arbitrarily change the height (Y) of the left part of the sample by a factor of k, then equation (11) is not true. Now we change the sides of the right part (X_0) by a factor of K in such a way that the equality becomes valid again. This time K is not arbitrary but is a function of k; then,

$$X^pF_1(kY/X) = K(k)^pX_0^pF_1(1). \quad (13)$$

If we take into account equation (12), then we have

$$F_1(kY/X) = K(k)^pF_1(Y/X). \quad (14)$$

The definition of the homogeneous function is as follows [16]: $F(kz) = g(k)F(z)$, where k is an arbitrary factor, and in general, $g(k)$ is power function: $g(k) = k^q$ [16]; therefore, $F(kz) = k^qF(z)$. In our case, we see that $K(k)^p = g(k) = k^q$ and

$$F_1(kY/X) = k^qF_1(Y/X).$$

So $F_1(Y/X)$ is a homogeneous function of its relative argument with exponent q. This expression must be valid for any k (including $k = X/Y$); therefore, $F_1(Y/X) = (Y/X)^qF_1(1)$. X and Y in the expression for the critical current are equivalent. Let $F_1(1) = A$. Using equation (5) we find that $q = p/2$ and $I_c(X, Y, T)$ is given by equation (10).

Equation (10) includes the product of X and Y, i.e. the area of the cross-section S; therefore we have

$$I_c(X, Y, T) = Af(T)S^{p/2}. \quad (15)$$

Figure 10 shows the dependence of the critical current of sample 3 on the cross-sectional area at various temperatures. In this sample a random Josephson net at temperatures rather more 70 K is formed by the grains of the superconducting phase Bi-2223. At lower temperatures, the grains of the two phases Bi-2223 and Bi-2212 produce different Josephson nets. It is important to note that the curves corresponding to higher temperatures (triangular signs ◁) are the same as the curves corresponding to lower temperatures (triangular signs ▶). The curves are similar to each other and may be approximated by

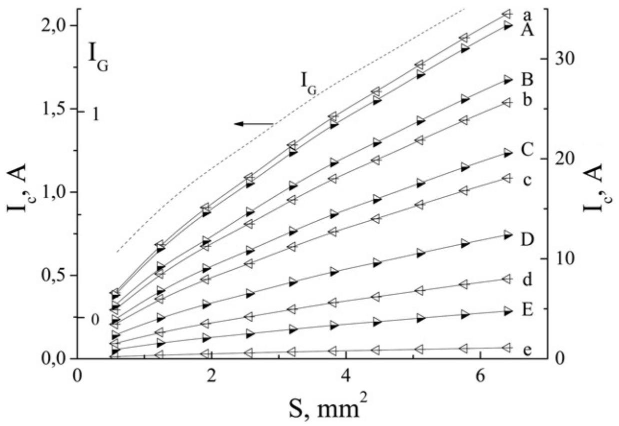

Figure 10. Critical current in sample 3 at various temperatures as a function of the cross-sectional area. Lines A–E correspond to isotherms (right scale) at 4.2, 22.4, 37.6, 53.8, 68.0 K. Lines a–e correspond to isotherms (left scale) at 73.7, 77.33, 81.0, 86.8, 93.3 K. The curve I_G (left scale) shows the run of the relative current $I_G = I_c(S)/I_c(S_0)$. $S_0 = 3.2$ mm^2.

equation (15) independently of the temperature range. The dashed line $I_G = I_c(S)/I_c(S_0)$ in figure 10 demonstrates that I_G is independent of the temperature and any specific properties of the Josephson net. Curves $I_c(S)$ and $I_G(S)$ for samples 1 and 2 are identical to $I_c(S)$ and $I_G(S)$ for sample 3. And so the function $I_G(S)$ does not depend on the temperature, or on the type of the Josephson net. Evaluating the exponent p for sample 3 for each isotherm, we obtain a set of values coinciding within confidence intervals. The deviations of the value of the exponent from the average value are of random nature, i.e., the exponent is independent of temperature as well as the type of random Josephson medium existing in the sample at a certain temperature. Index p is equal to 1.38 ± 0.02. We had shown earlier [15] that $I_c \sim R^p$ for samples having round cross-section (R is the radius of the cross-section), i.e. equation (15) is valid in this case too. For samples 1, 2, and 3 the average values of exponent p are equal to 1.37 ± 0.01, 1.36 ± 0.02, and 1.38 ± 0.02 respectively. These values coincide within confidence intervals with each other and with the earlier obtained value ($p = 1.36 \pm 0.01$ [15]). We find the weighted average of averages $p = 1.37 \pm 0.02$. Summarizing the results obtained here and earlier [14, 15] we may say that the transport critical current in a random three-dimensional Josephson net (namely, function $G(X, Y)$) is a homogeneous function of the size or area of the cross-section with exponent p. This exponent is a universal constant, i.e. it is independent of the net type, temperature, cross-sectional shape and so on. Function $G(X, Y)$ determines the critical state in the system. It is quite possible that factor A in equation (10) and (15) depends on the shape of the sample cross-section. Introducing a relative current we eliminate the specific properties of the sample, i.e. function $f(T)$, and multiplier A is eliminated at the same time.

We have shown that the critical current in a random three-dimensional Josephson net is a homogeneous function of the cross-sectional size for any cross-sectional shape:

$$I_c(kX, kY, T) = k^p I_c(X, Y, T). \tag{16}$$

Now we will investigate properties of the critical current density in such a system using this experimental fact. Let us introduce a system of orthogonal coordinates. For such cross-sections as circles, rectangles, ellipses and others that are important for investigations and practice, zero must be situated in the center of the geometrical figure. Let the domains representing our cross-sections be Ω_1 and Ω. In domain Ω_1 point x_1, y_1 corresponds to point x, y in Ω. Ω_1 and Ω are similar domains and so $x_1 = kx$ and $y_1 = ky$. Equation (16) may be rewritten as

$$\int_{\Omega_1}\int j_c(x_1 y_1)\,dx_1\,dy_1 = k^p \int_{\Omega}\int j_c(xy)\,dx\,dy. \tag{17}$$

Here j_c is a critical current density. We consider the infinitesimal area dS (dS = dxdy) as a physical infinitesimal area. Its dimensions must be much less than the sample cross-section dimensions or the dimensions of any domain under consideration, but its dimensions are much greater than an average cell dimension of the Josephson net, i.e. dS contains a great number of weak links. In other words, from our point of view the Josephson medium is a continuum. The change of variables in the integration in the left part of equation (17) gives

$$\int_{\Omega}\int \left(j_c(kx, ky) - k^{p-2} j_c(x, y)\right) dx\,dy = 0.$$

It was shown that equation (16) must be valid for any shape of cross-section (for any domain Ω); hence

$$j_c(kx, ky) = k^{p-2} j_c(x, y). \tag{18}$$

The critical current density in any random three-dimensional Josephson net is a homogeneous function of the coordinates and its index is equal to $p - 2$.

Let us consider the immediate corollaries.

(1) If ω_1 and ω are closed similar domains in the interior of the cross-section and any point x_1, y_1 in ω_1 corresponds to a point x, y in ω, then the critical current in ω_1 may be written as

$$i_c(\omega_1) = k^p i_c(\omega). \tag{19}$$

So the critical currents in similar domains differ by a factor k^p. If ω_1 coincides with the sample cross-section, then

$$I_c = k^p i_c(\omega). \tag{20}$$

(2) Writing the homogeneous function j_c in the form of equation (5) and using polar coordinates we have

$$j_c = r^{p-2} j_1(\varphi).$$

If the boundary curve of a domain situated on the cross-section and the cross-section boundary are homothetic figures, then the critical current density distribution on the curve is the same as that on the cross-section boundary. For a cross-section having piecewise-smooth boundary the statement may be made for each sector bounded with a smooth curved boundary. Since $p = 1.37$, then $j_c \sim r^{-0.68}$. The critical current density goes to infinity as r tends to zero. It must be noted that we regard a three-dimensional Josephson net as a continuum and our consideration cannot be used as r tends to a grain size.

(3) The critical current density is a homogeneous function and satisfies the Euler differential equation [16]:

$$x\partial j_c/\partial x + y\partial j_c/\partial y = (p-2)j_c. \tag{21}$$

Therefore, the critical current may be written as $I_c = (1/(p-2))\int\int_\Omega (x\partial j_c/\partial x + y\partial j_c/\partial y)\mathrm{d}x\mathrm{d}y$. If L is a cross-section boundary curve, then transforming the double integral to an integral over one cycle we have

$$I_c = \frac{1}{p}\oint_L x j_c\,\mathrm{d}y - y j_c\,\mathrm{d}x. \tag{22}$$

The value of the transport critical current in a random three-dimensional Josephson net is determined by values of the critical current density on the cross-section boundary curve. If ω is a closed domains in the interior of the cross-section and M is its boundary curve, then

$$i_c(\omega) = \frac{1}{p}\oint_M x j_c\,\mathrm{d}y - y j_c\,\mathrm{d}x. \tag{23}$$

Let us go into the question of the magnetic field induced in a random Josephson net by the transport critical current. Let us consider domains Ω_1 and Ω. We write

$$\frac{\partial H_y(x_1,y_1)}{\partial x_1} - \frac{\partial H_x(x_1,y_1)}{\partial y_1} = j_c(x_1,y_1) \tag{24}$$

$$\frac{\partial H_y(x,y)}{\partial x} - \frac{\partial H_x(x,y)}{\partial y} = j_c(x,y). \tag{25}$$

Changing variables in equation (24) and using equations (18) and (25) we obtain

$$\frac{\partial}{\partial x}\left[H_y(kx,ky) - k^{p-1}H_y(x,y)\right] - \frac{\partial}{\partial y}\left[H_x(kx,ky) - k^{p-1}H_x(x,y)\right] = 0.$$

This equation must be valid for any point of any domain Ω at any k. This is possible if the expressions in square brackets are equal to zero at the same time:

$$H_x(kx,ky) = k^{p-1}H_x(x,y) \tag{26}$$

$$H_y(kx,ky) = k^{p-1}H_y(x,y) \tag{27}$$

$$H(kx,ky) = k^{p-1}H(x,y). \tag{28}$$

Hence the value of the magnetic field induced by the transport critical current in a random three-dimensional Josephson net and the components of the field are homogeneous functions of the coordinates. The exponent equals $p-1$. The value of the critical current flowing through any domain Ω may by written as $I_c = \int\int_\Omega j_c\,\mathrm{d}x\,\mathrm{d}y$ and $I_c = \oint_L H\mathrm{d}l$. In these expressions the integration procedure does not deal with a temperature variable. So the expressions for the critical current density and the magnetic field contain the multiplier $f(T)$. As before, we can say that the magnetic field distribution on a curve situated in the cross-section and homothetic to the cross-sectional shape is the same as that on the boundary of the cross-section. The value of the magnetic field and its components go to zero as r tends to zero. All values describing the critical state in the random net obey the scaling law.

The distribution and value of the critical current density in a granular system depend on the magnetic field. Bean [17, 18], Kin and Anderson [19, 20] developed models for hard superconductors; these and some other models [21, 22] have long been used in order to clarify the effect of magnetic field on the critical current density in ceramic superconductors [4, 23–25]. This picture is useful but it can hardly be said to account completely for the remarkable properties of HTSC ceramics.

The critical current density depends on the magnetic field: $j_c = j_c(H)$. On the other hand the transport critical current density satisfies the Euler differential equation (21) and the magnetic field for its part satisfies the Euler differential equation containing the multiplier $p-1$. So

$$\begin{aligned} \mathrm{d}\ln j_c &= \frac{p-2}{p-1}\mathrm{d}\ln H \\ j_c(H) &= QH^{\frac{p-2}{p-1}}. \end{aligned} \tag{29}$$

The critical current density is a power function of the magnetic field value: $j_c \sim H^{-1.7}$. It is clear that the integration constant Q depends on the sample cross-sectional shape and its overall dimensions. Q depends on the temperature as $f(T)^{1/p-1}$.

4. Conclusion

The transport critical current in any random three-dimensional Josephson net existing in an HTSC ceramic can be presented as the product of the temperature and size dependent factors. The temperature factor $f(T)$ depends on temperature and specific properties of a sample material. The size factor $G(X,Y)$ contains information on the critical state in a Josephson net. The transport critical current depends on the cross-sectional area, independently of the cross-sectional shape (accurate within factor A). Values describing the critical state in a 3D Josephson net satisfy the scaling law. The critical current (namely, $G(X,Y)$) is a homogeneous function of the cross-sectional size or area with exponent p, $p = 1.37 \pm 0.02$. The exponent is a universal constant. It does not depend on temperature, specific sample properties, Josephson net type, cross-sectional shape, and so forth. The critical current density and induced magnetic field are homogeneous functions of the coordinates. The exponents of the functions are $p-2$ and $p-1$. The common multiple $f(T)$ determines the temperature dependence of the critical current, its density, and the induced magnetic field. The current value depends on the values of the critical current density on the cross-section boundary curve. The critical current density is a power function of the magnetic field.

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